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Question 1

For each of the following, evaluate y' at the indicated point. Give exact values (not calculator values) in simplified form for each.

a) $y = (2+x)^{3x} \; at \; x = 3$

Part-a

Get the natural logarithm of both sides

$$\ln y = 3x \ln(2+x)$$

Differentiate both sides

$$\frac{1}{y}\frac{dy}{dx} = 3x \cdot \frac{1}{2+x} + 3 \cdot \ln(2+x)$$

So

$\frac{dy}{dx} = y\left[\frac{3x}{2+x} + 3\ln(2+x)\right]$ or

$$\frac{dy}{dx} = (2+x)^{3x}\left[\frac{3x}{2+x} + 3\ln(2+x)\right]$$

At $x = 3$,

$$\frac{dy}{dx} = (2+3)^{9}\left[\frac{9}{2+3} + 3\ln(2+3)\right] = 9(5^{8}) + 3(5^{9})\ln 5 = 3515625 + 5859375 \ln 5$$

Part-b

$$\text{b) } y = \frac{(5x^2+1)^2(x+2)^5}{2x^2\left(\sqrt[3]{x^2+9}\right)}, \ at\ x = -3$$

The natural logarithm of both sides is

$$\ln y = 2\ln(5x^2+1) + 5\ln(x+2) - 4\ln x - \frac{1}{3}\ln(x^2+9)$$

Differentiate both sides

$$\frac{1}{y}\frac{dy}{dx} = \frac{2}{5x^2+1}(10x) + \frac{5}{x+2} - \frac{4}{x} - \frac{1}{3}\cdot\frac{1}{x^2+9}(x)$$

$$\frac{1}{y}\frac{dy}{dx} = \frac{20x}{5x^2+1} + \frac{5}{x+2} - \frac{4}{x} - \frac{x}{3(x^2+9)}$$

So

$$\frac{dy}{dx} = y\left[\frac{20x}{5x^2+1} + \frac{5}{x+2} - \frac{4}{x} - \frac{x}{3(x^2+9)}\right]$$ or

$$\frac{dy}{dx} = \frac{(5x^2+1)^2(x+2)^5}{2x^2\sqrt[3]{x^2+9}}\left[\frac{20x}{5x^2+1} + \frac{5}{x+2} - \frac{4}{x} - \frac{x}{3(x^2+9)}\right]$$

At $x = -3$,

$$\frac{dy}{dx} = \frac{-(45+1)^2}{18\sqrt[3]{9+9}}\left[\frac{-60}{45+1} - 5 + \frac{4}{3} + \frac{3}{3(9+9)}\right]$$

$$= \frac{-46^2}{18\sqrt[3]{3^2(2)}}\frac{\sqrt[3]{2^2(3)}}{\sqrt[3]{2^2(3)}}\left[\frac{-60}{46} - 5 + \frac{4}{3} + \frac{1}{18}\right]$$

$$= \frac{-46^2}{18(6)}\left(\sqrt[3]{12}\right)\left(\frac{-2035}{414}\right)$$

$$= \frac{46805}{486}\sqrt[3]{12}$$ which is approximately 220.487

Part-c

c) $2^{x+5}\log_2\left(4x^3\right)$ *at* $x = 2$

We can simplify to

$y = 2^{x+5}\left(\log_2 4 + 3\frac{\ln x}{\ln 2}\right)$ or

$y = 32(2^x)\left(2 + \frac{3\ln x}{\ln 2}\right)$

Differentiate using product rule

$$\frac{dy}{dx} = 32(2^x)(\ln 2)\left(2 + \frac{3\ln x}{\ln 2}\right) + 32(2^x)\left(\frac{3}{x\ln 2}\right)$$

At $x = 2$,

$$\frac{dy}{dx} = 32(4)(\ln 2)\left(2 + \frac{3\ln 2}{\ln 2}\right) + 32(4)\left(\frac{3}{2\ln 2}\right)$$

$$= 640\ln 2 + \frac{192}{\ln 2}$$

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Question 2

A certain population of squirrels is represented by the function $P(t) = 3t\left(e^{-\frac{t}{5}}\right)$, where P is the number of squirrels measured in hundreds after t weeks.

a) Determine the maximum number of squirrels present in the population.

Part-a

Differentiate

$$P'(t) = 3\left(e^{-t/5}\right) + 3t\left(\frac{-1}{5}e^{-t/5}\right)$$

Equate the derivative to 0 and solve for t

$$3\left(e^{-t/5}\right) + 3t\left(\frac{-1}{5}e^{-t/5}\right) = 0$$

Divide both sides by $e^{-t/5}$

$$3 - \frac{3t}{5} = 0$$

$$t = 5$$

$P(5) = 15e^{-1}$ multiply this by 100

Approximately 551 or 552 squirrels are present in the population after 5 weeks

Part-b

b) Does the squirrel population ever die out? Explain.

The function $P(t) = 3te^{-t/5}$ cannot be zero for t > 0

However, as $t \to \infty$, $P(t) \to 0$

So after a very large number of weeks, the population will approach zero.

 # 3

Question 3

For what values of k does $y = Ae^{kt}$ satisfy the following equation, when A and k are constant? Explain your steps.

y" + 6y' = 40y

The first derivative is

$y' = Ake^{kt}$

The second derivative is

$y'' = Ak^2e^{kt}$

Therefore, the left side is

$y'' + 6y' = Ak^2e^{kt} + 6Ake^{kt}$

and the right side is

$40y = 40Ae^{kt}$

Equating the left and right side, we have

$Ak^2e^{kt} + 6Ake^{kt} = 40Ae^{kt}$

$(k^2 + 6k)Ae^{kt} = 40Ae^{kt}$

Divide both sides by Ae^{kt}

$k^2 + 6k = 40$

$k^2 + 6k - 40 = 0$

$(k + 10)(k - 4) = 0$

The values of k are $k = -10$ or $k = 4$